

THE FIRST LUNAR LASER RANGING CONSTRAINTS ON GRAVITY SECTOR SME PARAMETERS

J.B.R. BATTAT, J.F. CHANDLER AND C.W. STUBBS

*Harvard-Smithsonian Center for Astrophysics
60 Garden Street,
Cambridge, MA 02138, USA
E-mail: jbattat@cfa.harvard.edu*

We present the first constraints on pure-gravity sector Standard-Model Extension (SME) parameters using Lunar Laser Ranging (LLR). LLR measures the round trip travel time of light between the Earth and the Moon. With 34+ years of LLR data, we have constrained six independent linear combinations of SME parameters at the level of 10^{-6} to 10^{-11} . There is no evidence for Lorentz violation in the LLR dataset.

1. Introduction and Motivation

Two of us (C.W.S. and J.B.R.B.) are members of the Apache Point Observatory Lunar Laser-ranging Operation (APOLLO), a next-generation LLR facility, capable of millimeter-precision lunar range measurements (see the article by T.W. Murphy in these proceedings). The APOLLO project was motivated by the realization that an order-of-magnitude improvement in fundamental physics constraints (e.g. equivalence principle, gravitomagnetism, gravitational $1/r^2$ law and \dot{G} , to name a few) could be achieved with straightforward improvements to the standard LLR apparatus.

With the recent description of the pure-gravity sector of the SME,¹ we learned that LLR can also provide incisive constraints on Lorentz Violation. The predicted LLR observable under Lorentz Violation is a periodic perturbation to the Earth-Moon range with the leading order effects occurring at four distinct frequencies: 2ω , ω , $2\omega - \omega_0$ and Ω_\oplus . Here ω is the lunar orbital (sidereal) frequency, ω_0 is the anomalistic lunar orbital frequency and Ω_\oplus is the mean Earth orbital (sidereal) frequency. Although the APOLLO program is still in the data collection phase, there are more than three decades of freely-available archival LLR data on a public archive.² In this article, I present the SME parameter constraints that result from our anal-

ysis of archival LLR data. These are the first LLR-based constraints on pure-gravity SME parameters.

2. The LLR Dataset and Analysis Software

LLR measures the time of flight of photons between a telescope on the Earth and corner cubes on the lunar surface.³ LLR data is typically presented in “normal points” which are typically generated from a few to a hundred lunar signal photons collected over a span of 1 to 5 minutes. Our analysis makes use of archival data from September 1969 through December 2003.

In the analysis of LLR normal points, a set of residual ranges is computed by subtracting the model’s predicted range from the observed range. The range sensitivity with respect to each model parameter (the partial derivatives) is also computed at the time of each normal point. The residuals and the partial derivatives are then used to compute optimal model parameter values via a weighted linear least-squares fit. For our analysis, we used the Planetary Ephemeris Program (PEP),⁵ which is currently maintained by one of us (J.F.C.). To our knowledge, it is the only publicly available LLR analysis software.

Typically, the lunar range model is formulated in the parametrized post-Newtonian (PPN) framework,⁴ which permits model-independent constraints on metric theories of gravity. At present, no ephemeris models explicitly incorporate SME parameters. You can, however, think of these models as implicitly including the SME parameters but with values pegged at zero (*i.e.* no SME perturbation to the lunar orbit). It is therefore only necessary to compute, by hand, the partial derivative of range with respect to each SME model parameter (see Table 1). The analysis code can then provide SME parameter adjustments simultaneously with the other model parameters. A covariance matrix including the correlations between the SME parameters and all other model parameters is also produced.

The main drawback to this approach is that one cannot perform an iterative analysis in which one takes the best-fit model parameter values and uses them to re-integrate the equations of motion to refine the model parameter values. We accept this limitation because the non-SME model parameters have been highly refined through iterative solutions over the past several decades and so the solution sits firmly in the linear regime already. Furthermore, the addition of the SME parameters preserves the linearity because the lunar range is strictly linear in the SME parameters (see Table 2 of Ref. 1), so no iteration is necessary.

Table 1. SME parameter partial derivatives. Symbols used here are explained in Ref. 1.

SME Parameter	Partial Derivative of Lunar Range with Respect to SME Parameter
$\bar{s}^{11} - \bar{s}^{22}$	$-\frac{r_0}{12} \cos(2\omega t + 2\theta) - \frac{\omega e r_0}{16(\omega - \omega_0)} \cos[(2\omega - \omega_0)t + 2\theta]$
\bar{s}^{12}	$-\frac{r_0}{6} \sin(2\omega t + 2\theta) - \frac{\omega e r_0}{8(\omega - \omega_0)} \sin[(2\omega - \omega_0)t + 2\theta]$
\bar{s}^{02}	$-\frac{\omega(\delta m)v_0 r_0}{M(\omega - \omega_0)} \cos(\omega t + \theta)$
\bar{s}^{01}	$\frac{\omega(\delta m)v_0 r_0}{M(\omega - \omega_0)} \sin(\omega t + \theta)$
$\bar{s}_{\Omega_{\oplus},c}$	$V_{\oplus} r_0 \left(\frac{b_1}{b_2} \right) \cos(\Omega_{\oplus} t)$
$\bar{s}_{\Omega_{\oplus},s}$	$V_{\oplus} r_0 \left(\frac{b_1}{b_2} \right) \sin(\Omega_{\oplus} t)$

3. Systematic Errors

The solar system is complex. Predictions of the lunar range rely on models of planetary and asteroid positions, gravitational harmonics of the Sun, Earth and Moon and various relativistic and non-gravitational effects (to name a few). Solar system models have many hundreds of parameters that influence the Earth-Moon range time. There are strong correlations between model parameters. As a result, solutions will suffer from systematic errors in model parameter estimates that can dominate the formal errors reported by the least-squares analysis. In this work, we account for the underestimation of model parameter uncertainties by scaling the formal parameter errors reported by the least-squares analysis by a uniform factor, F . This is numerically equivalent to uniformly scaling the uncertainty of each normal point by F . Essentially, we uniformly down-weight the data.

The F factor is empirically determined by holding the SME parameter values at zero but allowing the PPN values β and γ to vary (see Ref. 4 for an explanation of β and γ). We know from existing experiments⁶ that these parameters are consistent with their General Relativity values ($\beta = \gamma = 1$) to within a part in 10^3 or better. We find that we require $F = 20$ to ensure that we are in accord with these earlier results.

4. SME Parameter Constraints and Verification

We constrain the SME parameters under the assumption that General Relativity is not violated (we set $\beta = \gamma = 1$). The resulting parameter estimates and their realistic errors (the formal errors scaled by $F = 20$) are reported in Table 2. All SME parameters are within $1.5F\sigma$ of zero. There is no evidence for Lorentz violation in the LLR data. The fit quality is shown in Fig. 1.

To verify our implementation of the partial derivatives of lunar range with respect to the SME parameters, we generated, by hand, a perturbed LLR normal point data set by setting $\bar{s}^{11} - \bar{s}^{22} = 9 \times 10^{-10}$, a $10F\sigma$ deviation

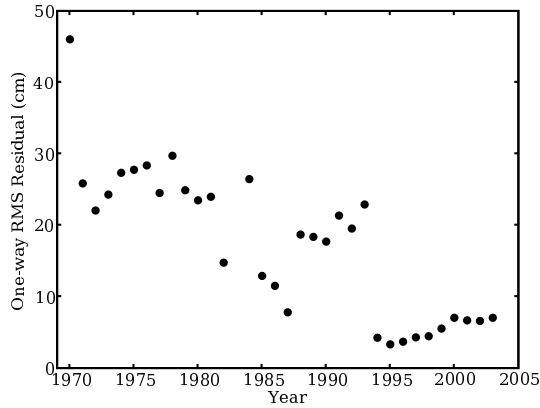


Figure 1. The agreement between model and data for the first 34 years of LLR.

from the best-fit value of this parameter. A fit to this data recovers the perturbation: $\bar{s}^{11} - \bar{s}^{22} = [(1 + 9) \pm 0.9] \times 10^{-10}$ with the other SME parameters unchanged.

5. Conclusions and Future Prospects

We have analyzed 34+ years of LLR data and have derived constraints on six SME parameters combinations (see Table 2). We find no deviation from Lorentz Symmetry at the $10^{-6} - 10^{-11}$ level. This work provides the first LLR-based constraints of SME parameters.

There are several ways in which these constraints could be improved. First of all, by incorporating auxiliary solar system data (e.g. planetary radar ranging) model parameter correlations can be reduced, and F decreased. This would allow for tighter constraints on the SME parameters using the same LLR dataset. In addition, APOLLO data, which is about 10 times more precise than the archival data, will soon be ready for analysis. With this improved dataset, we will further tighten the SME parameter constraints.

Acknowledgments

This work developed from discussions with our colleagues including E. Adelberger, Q. Bailey, J. Davis, A. Kostelecký, J. Moran, T. Murphy, I. Shapiro and M. Zaldarriaga. We would also like to thank the staff observers at

Table 2. SME parameter estimates and their realistic (scaled) uncertainties ($F\sigma$) with $F = 20$.

Parameter	Estimate
$\bar{s}^{11} - \bar{s}^{22}$	$(1.3 \pm 0.9) \times 10^{-10}$
\bar{s}^{12}	$(6.9 \pm 4.5) \times 10^{-11}$
\bar{s}^{02}	$(-5.2 \pm 4.8) \times 10^{-07}$
\bar{s}^{01}	$(-0.8 \pm 1.1) \times 10^{-06}$
$\bar{s}_{\Omega \oplus c}$	$(0.2 \pm 3.9) \times 10^{-07}$
$\bar{s}_{\Omega \oplus s}$	$(-1.3 \pm 4.1) \times 10^{-07}$

LLR stations, and the ILRS for the infrastructure that supports the normal point distribution. J.B.R.B. acknowledges financial support from the ASEE NDSEGF, the NSF GRFP and Harvard University. We also acknowledge the generous financial support of the National Science Foundation (grant PHY-0602507).

References

1. Q.G. Bailey and V.A. Kostelecký Phys. Rev. D **74**, 045001 (2006);
2. M.R. Pearlman *et al.*, Advances in Space Research **30**, 135 (2002).
3. J.G. Williams *et al.*, gr-qc/0507083.
4. C.M. Will and K.J. Nordtvedt, Astrophys. J. **177**, 757 (1972).
5. The Planetary Ephemeris Program is a solar system ephemeris and data analysis program that was developed at the Massachusetts Institute of Technology and its Lincoln Laboratory, beginning in 1964. The source code is currently maintained by John Chandler at the Center for Astrophysics in Cambridge, MA and is publicly available.
6. J.G. Williams *et al.*, Phys. Rev. Lett. **93**, 261101 (2004); B. Bertotti *et al.*, Nature (London) **425**, 374 (2003); S.S. Shapiro *et al.*, Phys. Rev. Lett. **92**, 121101 (2004); J.F. Chandler *et al.*, in *Proc. of the Seventh Marcel Grossman Meeting*, R.T. Jantzen *et al.*, eds., (1996).
7. See, for example, V.A. Kostelecký, ed., *CPT and Lorentz Symmetry III*, World Scientific, Singapore, 2005.
8. J. Lipa *et al.*, Phys. Rev. Lett. **90**, 060403 (2003); P.L. Stanwix *et al.*, Phys. Rev. D **74**, 081101 (2006); M.E. Tobar *et al.*, Phys. Rev. D **71**, 025004 (2005); M. Hohensee *et al.*, Phys. Rev. D **75**, 049902 (2007); J.P. Cotter and B. Varcoe, physics/0603111; H. Müller *et al.*, arXiv:0706.2031.
9. T. Katori and R. Tayloe, in Ref. 7; LSND Collaboration, L.B. Auerbach *et al.*, Phys. Rev. D **72**, 076004 (2005); V.A. Kostelecký and M. Mewes, Phys. Rev. D **69**, 016005 (2004).